Predator and Prey: The mathematics of a balanced ecosystem

STOCK PHOTO OF SNOW LYNX AND SNOW HARE

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Abstract

Introduction

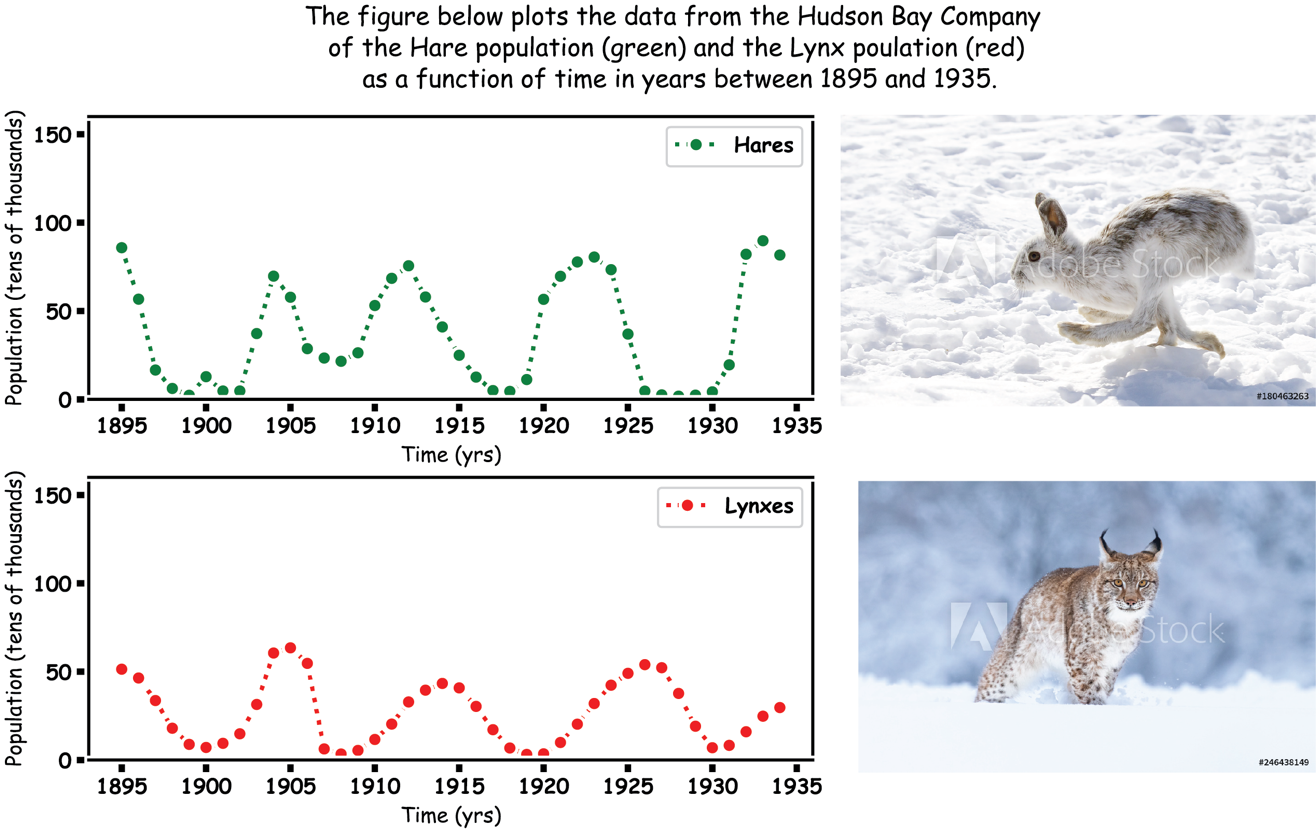
Understanding how animal populations change over time helps with the preservation of animals and how animals interact with their environment. Investigating and modeling the relationship between predators (animals that hunt and eat other animals) and prey (the animal that is food for the predators) helps use understand the natural ups and downs of a population to when it could be at risk of extinction.

Interestingly one of the first observations of this up and down was by a company that was hunting both the predator and prey for their fur in the 19th and 20th century. The Hudson Bay Company made yearly records of the amount of snow lynx and snowshoe hare pelts they collected. Figure 1 shows data for the number of hare pelts in green over time top row and the Lynx pelts over time in red bottom row. The data showed that some years there were more lynxes (predators) but less hares (prey) like 1927 while other years, like 1932 there were more hares but less lynxes.

The up and down in the hare and lynx population over time makes sense as it suggested there might be a relationship between the two, the lynxes eats the hares. When there are more lynxes they eat more hares which decreases the hare population. But when the hare population is low it is hard for the lynxes to find the hares which means less food and results in a decrease in the lynx population. When, the lynx population decreases the hare population will increase again and the cycle will continue. If the populations are balanced they will go up and down over time. Up and down like waves.

abundance of prey, the predators will eat more and the prey population will increase. On the other hand this will lead to a shortage in the number of prey, so it will decrease the number of predators [1-4], which will give the a chance for the prey to grow in population.

This up down cycle will keep continuing in a balanced eco-system.



Mathematical Models

Mathematicians use equations to describe this aktan cycle my eyes can describe had two animals interact and it can predict what should happen

One thing is to observe the data but with mathematics we can model the data which can be used to make predictions and model if something changes in the environment.

In this paper we will show how mathematics can be used to model the populations of Lynxes and Hares.

The model was first introduced in 1920s by Lotka (Lotka, 1920) and Volterra (Volterra, 1926), the both noticed this up and down and they wished to see if they could use mathematics to explain what people seen in the natural world.

**Lotka and Volterra used differential equations to model animals. Now, when you first come across the differential equations it can look very complicated but all they are is a way of say the change in population is impacted by different things. A famous mathematician called Euler said that differential equations could just be written as plusses and minuses.**

***Hares***

Before we look at the mathematics let’s think about what is happening to the Hares.

The future Hare population depends on the number of hares and the number of lynxes.

We just use + to add to the population and – takes away from the population. It is as simple as that. If there were no Lynxes than the Hare population in the future is just equal to the current hare population plus births minus deaths.

The future population of the Hares, which in maths we will call H\_Future, is calculated by adding the current population plus the number of new hares minus the number of hares the lynx population eats

So this can be written as a the mathematics equation using plusses and minuses

this can be actually rewritten In a much simpler form ask the current population Is equal to the past population Plus time times the impact of prey upon predator which is a negative it's subtraction So what we have here is just addition Multiplication and some subtraction

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what actually happening there is a change in the prey population That is due to then mega prayer is and how many pray the predator can eat that is our first equation in words

Lynxes

Well for the predator if there is no food source it would just dwindle in size so well we actually have is the change in predator population is equal to the current predator population Adding pray in predator interaction

This can be rewritten as the future predator population is Is equal to current predator population Plus time times the interaction Of prey and predator which is an addition

So what we have here is just plus minus plus minus and that's how to mathematics described the Uptown relationship of predators and praise

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The Meaning

Further Implications

now the one thing that you should note when mathematicians try to describe something complicated they try to simplify it so here are some simplifications we have to make 1 these animals live on an island where there are no other predators and no other prey 2 the rabbits have an abundance of food so we don't have to worry about them fresh 3 there is no external input to the island add Dave my team like city oversimplifications but even taking them into account we can still do really really nice models to let's begin.

You need good data to get the numbers to start the model, which means scientist, conservations and even fur hunters collecting the information.

There are some problems with these equations, such as

1. There are way more than one predator of the snow hare, what about snow foxes.
2. Lynxes have a bigger diet than just snow hares, they can also eat some birds to survive.
3. What if the hares eat up all the grass, then they will have no food
4. What about human fur hunters, who hunt both lynxes?

To make the equations work for all these questions you can include extra equations and pluses and minuses. If you have all the data you could perfectly model the future.

The models can also be used to predict the impact or re-introducing a previous extinct animal back into an area like Jurassic Park or how wolves were re-introduced into Yellowstone park with surprising results watch the video []. There were concerns that the re-introduction of wolves would mean that there would be a decrease in the deer and other animals. But it was just the opposite, when the wolves came there was an increase in the population of all the prey and even more because the wolves scared the deer and other animals away from the river bank which mean the plants could flourish again which meant more food for the deer which mean more deer. This was surprising but if they had a model they could of predicted this amazing increase in all the animal population [5].

This predator prey relationship can be expanded further outside of just animals and can be use to model how companies interact, chemical reactions, and how viruses spread.

You can read more about this in another frontiers paper for young minds NAMES which talks about using similar mathematics to model and understand the spread of COVID 19.

Conclusion

You can see

Open Questions and Future Discussions

The model shows the benefit of wolves to a simple ecosystem model of plants and deer. Mathematical models of this type can be used to motivate the discussion.

The immediate questions that can arise from this simulation are:

How many wolves are beneficial?

Does Ireland have the same ecosystem as yellowstone?

What are hidden negatives?

What are hidden positives?

You can also look at the notebook on the negative impact of microplastics to our ecosystem [4].

References

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